UNCERTAINTY IN THE GLOBAL AVERAGE SURFACE AIR TEMPERATURE INDEX: A REPRESENTATIVE LOWER LIMIT

by

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UNCERTAINTY IN THE GLOBAL AVERAGE SURFACE AIR TEMPERATURE INDEX: A REPRESENTATIVE LOWER LIMIT

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ABSTRACT
Sensor measurement uncertainty has never been fully considered in prior appraisals of global average surface air temperature. The estimated average ±0.2 C station error has been incorrectly assessed as random, and the systematic error from uncontrolled variables has been invariably neglected. The systematic errors in measurements from three ideally sited and maintained temperature sensors are calculated herein. Combined with the ±0.2 C average station error, a representative lower-limit uncertainty of ±0.46 C was found for any global annual surface air temperature anomaly. This ±0.46 C reveals that the global surface air temperature anomaly trend from 1880 through 2000 is statistically indistinguishable from 0 C, and represents a lower limit of calibration uncertainty for climate models and for any prospective physically justifiable proxy reconstruction of paleo-temperature. The rate and magnitude of 20th century warming are thus unknowable, and suggestions of an unprecedented trend in 20th century global air temperature are unsustainable.

1. INTRODUCTION
The rate and magnitude of climate warming over the last century are of intense and continuing international concern and research [1, 2]. Published assessments of the sources of uncertainty in the global surface air temperature record have focused on station moves, spatial inhomogeneity of surface stations, instrumental changes, and land-use changes including urban growth.

However, reviews of surface station data quality and time series adjustments, used to support an estimated uncertainty of about ±0.2 C in a centennial global average surface air temperature anomaly of about +0.7 C, have not properly addressed measurement noise and have never addressed the uncontrolled environmental variables that impact sensor field resolution [3-11]. Field resolution refers to the ability of a sensor to discriminate among similar temperatures, given environmental exposure and the various sources of instrumental error.

In their recent estimate of global average surface air temperature and its uncertainties, Brohan, et al. [11], hereinafter B06, evaluated measurement noise as discountable, writing, "The random error in a single thermometer reading is about 0.2 C (1σ) [Folland,
et al., 2001] ([12]); the monthly average will be based on at least two readings a day throughout the month, giving 60 or more values contributing to the mean. So the error in the monthly average will be at most $0.2 / \sqrt{60} = 0.03$ C and this will be uncorrelated with the value for any other station or the value for any other month." Paragraph [29] of B06 rationalizes this statistical approach by describing monthly surface station temperature records as consisting of a constant mean plus weather noise, thus, "The station temperature in each month during the normal period can be considered as the sum of two components: a constant station normal value (C) and a random weather value (w, with standard deviation $\sigma$)." This description plus the use of a $1 / \sqrt{60}$ reduction in measurement noise together indicate a signal averaging statistical approach to monthly temperature.

1.1. The scope of the study
This study evaluates a lower limit to the uncertainty that is introduced into the temperature record by the estimated noise error and the systematic error impacting the field resolution of surface station sensors.

Basic signal averaging is introduced and then used to elucidate the meaning of the estimated $\pm0.2$ C average uncertainty in surface station temperature measurements as described by Folland, et al. [12]. An estimate of the noise uncertainty in any given annual temperature anomaly is then developed. Following this, the lower limits of systematic error in three temperature sensors are calculated using previously reported ideal field studies [13].

Finally, the average measurement noise uncertainty and the lower limit of systematic error in a Maximum–Minimum Temperature System (MMTS) sensor are combined into a total lower limit of uncertainty for an annual anomaly, referenced to a 30-year mean. The effect of this lower-limit uncertainty on the global average surface air temperature anomaly time series is described. The study ends with a summary and a brief discussion of the utility of the instrumental surface air temperature record as a validation target in climate studies.

2. SIGNAL AVERAGING
The error in an observable due to random noise can be made negligible by averaging repetitive measurements [14, 15]; a technique that is exploited to excellent effect in spectroscopy [16]. Three cases below show when noise reduction by signal averaging is appropriate, and when it is not. The statistical model in B06 is then appraised in light of these cases.

2.1. Case 1
In signal-averaging repetitive measurements of a constant temperature, the measurement in a random noise model is,

$$t_i = \tau_c + n_i$$

where $t_i$ is the measured temperature, $\tau_c$ is the constant "true" temperature, and $n_i$ is the
random noise associated with the \( i^{th} \) measurement. When the noise is stationary, it has a constant average intensity and a mean of 0. The mean temperature is \( \bar{T} = \frac{1}{N} \sum_{i=1}^{N} t_i \), and the ‘mean temperature ± mean noise’ is,

\[
\bar{T} \pm \bar{n} = \frac{1}{N} \sum_{i=1}^{N} t_i \pm \frac{1}{N} \sqrt{\sum_{i=1}^{N} (t_i - \bar{T})^2} .
\]  

(2)

When \( N \) is large and the noise is stationary, \( t_i - \bar{T} = n_i \) and \( \sqrt{\sum_{i=1}^{N} (t_i - \bar{T})^2} = \sqrt{\sum_{i=1}^{N} (n_i)^2} \Rightarrow N \sigma_n^2 \), where \( \sigma_n^2 \) is the variance of the noise intensity, and "\( \Rightarrow \)" signifies ‘approaches equality with.’ Finally,

\[
\bar{T} \pm \bar{n} = \frac{1}{N} \sum_{i=1}^{N} t_i \pm \frac{1}{N} \sqrt{N \sigma_n^2} \Rightarrow T \pm \frac{\sigma_n}{\sqrt{N}} .
\]  

(3)

That is, given a constant temperature and stationary random noise, averaging repetitive measurements of any constant temperature reduces the impact of noise as \( 1/\sqrt{N} \), and at large \( N \), \( \bar{T} \Rightarrow \tau \) and \( \sigma_n/\sqrt{N} \Rightarrow 0 \) [15, 17, p. 53ff]. Noise reduction by signal averaging is thus entirely appropriate when data fall within Case 1.

2.2. Case 2

Now suppose the conditions of Case 1 are changed so that the \( N \) true temperature magnitudes, \( \tau_i \), vary inherently but the noise variance remains stationary and of constant average intensity. Thus, \( \tau_1 \neq \ldots \neq \tau_i \neq \ldots \neq \tau_n \), while \( \sigma_i^2 = \ldots = \sigma_j^2 = \ldots = \sigma_n^2 \). Then,

\[
t_i = \tau_i + n_i ,
\]  

(4)

where \( t_i \) is again the measured temperature, \( \tau_i \) is the "true" instantaneous temperature, and \( n_i \) is again the noise intensity associated with the \( i^{th} \) measurement. This case may reflect a series of daily temperatures from any well-sited and maintained surface station sensor. In this case the ‘mean temperature ± mean noise’ will again be,

\[
\bar{T} \pm \bar{n} = \frac{1}{N} \sum_{i=1}^{N} t_i \pm \frac{1}{N} \sqrt{\sum_{i=1}^{N} (n_i)^2} = \bar{T} \pm \frac{\sigma_n}{\sqrt{N}} .
\]  

(5)
However, a further source of uncertainty now emerges from the condition $\tau_i \neq \tau_j$. The mean temperature, $\bar{T}$, will have an additional uncertainty, $\pm s$, reflecting the fact that the $\tau_i$ magnitudes are inherently different. The result is a scatter of the inherently different temperature magnitudes about the mean, because now $(t_i - \bar{T}) = (n_i + \Delta \tau_i)$, where $\Delta \tau_i$ represents the difference between the "true" magnitude of $\tau_i$ and $\bar{T}$, apart from noise. The magnitudes of the $n_i$ and the $\tau_i$ are physically independent and uncorrelated, and $n_i$ and $\Delta \tau_i$ are statistically independent. Therefore the uncertainties due to these factors can be calculated separately:

$$
\sigma^2_{\text{noise}} = \frac{1}{N} \sum_{i=1}^{N} (n_i)^2 \rightarrow \pm \frac{\sigma_{\text{noise}}}{\sqrt{N}}, \quad \text{but} \quad s^2 = \frac{1}{N-1} \sum_{i=1}^{N} (\Delta \tau_i)^2 \quad [17, \text{p. 9ff}] \quad (6)
$$

The condition of noise stationarity means that the $n_i$ have a true constant mean of zero (i.e., $\mu_{\text{noise}} = 0$). However, the second part of eqn. (6) shows that use of the empirical mean temperature, $\bar{T}$, in calculating $\Delta \tau_i = \tau_i - \bar{T}$, removes a degree of freedom from $s^2$. Following from eqns. (5) and (6), although the impact of random noise on $\bar{T}$ diminishes with $1/\sqrt{N}$, the magnitude uncertainty in $\bar{T}$, given by, $\pm s = \sqrt{\frac{\sum_{i=1}^{N} (\Delta \tau_i)^2}{N-1}}$ [17, p. 9ff], does not.

For Case 2 measurements the noise variance, $\sigma^2_n$, and the magnitude uncertainty, $\pm s$, must enter into the total uncertainty in the mean temperature as $\bar{T} \pm (\sigma_n/\sqrt{N}) \pm s$.

Therefore under Case 2, the uncertainty never approaches zero no matter how large $N$ becomes, because although $\pm \sigma_n$ should automatically average away, $\pm s$ is never zero.

The usual way to represent the uncertainty in averages of inherently varying magnitudes is with the standard deviation (SD) of the total scatter about the mean [17, p. 11], e.g., $SD = \sqrt{\frac{\sum_{i=1}^{N} (t_i - \bar{T})^2}{(N-1)}}$. If the sensor $\pm \sigma_n$ has been measured independently, then $\pm s$ can be extracted as $\pm s = \pm SD \pm (\sigma_n/\sqrt{N})$ because measurement noise and magnitude scatter are statistically independent. The magnitude uncertainty, $\pm s$, is a measure of how well a mean represents the state of the system. A large $\pm s$ relative to a mean implies that the system is composed of strongly heterogeneous sub-states poorly represented by the mean state [18]. This caution has bearing on the physical significance of mean temperature anomalies (see below).

2.3. Case 3

Finally, suppose a series of $N$ temperature measurements of inherently unique magnitudes but now also with unique and unequal noise variances. Thus as in Case 2,
This condition could arise from a time-dependent shift in the magnitude of the measurement noise of a single sensor, or when averaging temperatures from multiple sensors that each exhibits an independent and unique noise variance. The latter situation is closest to a real-world spatial average, in which temperature measurements from numerous stations are combined.

2.3.1. Case 3a
If the unequal noise variances from each and all of the station sensors are known to be stationary and uncorrelated, then \( T = \frac{1}{N} \sum_{i=1}^{N} t_i \) and the variance of the mean is \( \sigma_{\mu}^2 = \frac{1}{\sum_{i=1}^{N} \left(1/\sigma_i^2\right)} \) [17, p. 57]. One can simplify analysis by scaling all the variances to a single variance, thus \( \sigma_i^2 = \sigma_j^2 = \sigma_k^2 = \sigma_c^2 \), where \( c_j, c_k, \ldots, c_n \) are the coefficients of scale. Each of the N unequal variances entering a mean can now be transformed into an average variance of uniform magnitude as,

\[
\frac{1}{N} (\sigma_i^2 + c_j \sigma_i^2 + c_k \sigma_i^2 + \ldots + c_n \sigma_i^2) = \frac{1}{N} (1 + c_j + c_k + \ldots + c_n) \sigma_i^2 = q \sigma_c^2 = q \sigma_{\mu}^2
\]

where \( q = (1/N) \times (1 + c_j + c_k + \ldots + c_n) \), and \( \sigma_{\mu}^2 = \sigma_c^2 \) is a constant stationary variance. On combining N measurements, the variance of the mean becomes,

\[
\sigma_{\mu}^2 = \sqrt{\frac{\sum_{i=1}^{N} \left(1/\sigma_c^2\right)}{q^2}} = \sqrt{\frac{1}{q^2 N}} = \sqrt{\frac{1}{q^2 N}}
\]

The average noise uncertainty in \( T \) is then \( \pm \sigma_{\text{noise}} = \pm \sqrt{q \times (\sigma_c / \sqrt{N})} \), and if \( \sigma_i^2 \) was the minimum of variances, then \( \sqrt{q} > 1 \). Thus, with stationary noise variances of known but uneven magnitude, an average noise uncertainty can be found, \( \pm \sigma_{\text{noise}} = \pm \sqrt{q} \sigma_c \), that again diminishes as \( 1/\sqrt{N} \). It is important to notice that \( \sigma_{\mu}^2 \) of Case 2, a variance of noise, is calculationally and conceptually distinct from \( \sigma_n^2 \) of Case 3, an average of variances.

2.3.2. Case 3b
When sensor noise variances have not been measured and neither their stationarity nor their magnitudes are known, an adjudged average noise variance must be assigned using physical reasoning [19]. For multiple sensors of unknown noise provenance, or for a time series from a single sensor of unknown and possibly irregular variance, an
The adjudged estimate of measurement noise variance is implicitly a simple average,
\[ \bar{\sigma}_{\text{noise}}' = \frac{1}{N} \sum_{i=1}^{N} \sigma_i'^2, \]
where each \( \sigma_i'^2 \) is of unknown provenance and nominally represents
the unique noise variance of one of the \( N \) measurements. The primed sigma indicates
an adjudged estimate and distinguishes Case 3b noise uncertainty from those of Cases
1-3a.

In the case of an adjudged average noise uncertainty, each temperature
measurement must be appended with the constant uncertainty estimate as, \( t_i \pm \bar{\sigma}_{\text{noise}}' \).

The mean of a series of \( N \) measurements is the usual \( \bar{T} = \frac{1}{N} \sum_{i=1}^{N} t_i \), but the average
noise uncertainty in the measurement mean is \( \pm \bar{\sigma}_{\mu}' = \sqrt{N} \times \bar{\sigma}_{\text{noise}}' / (N - 1) \) \[17, p. 58, \]
with one degree of freedom lost because the estimated noise variance in each
measurement is an implied mean.

Thus when calculating a measurement mean of temperatures appended with an
adjudged constant average uncertainty, the uncertainty does not diminish as \( 1/\sqrt{N} \). Under Case 3b, the lack of knowledge concerning the stationarity and true magnitudes
of the measurement noise variances is properly reflected in a greater uncertainty in the
measurement mean. The estimated average uncertainty in the measurement mean,
\( \pm \bar{\sigma}_{\mu}' \), is not the mean of a normal distribution of variances, because under Case 3b the
magnitude distribution of sensor variances is not known to be normal.

The condition \( \tau_i \neq \tau_j \) in Case 3 also produces a magnitude uncertainty, \( \pm \bar{s} \), in
analogy with Case 2. When the magnitudes and stationarities of measurement noise
variances are both unknown, the total uncertainty in a measurement mean is
\[ \pm \bar{\sigma}_{\text{total}}' = \sqrt{\sum_{i=1}^{N} (t_i - \bar{T})^2 / (N - 1) }. \]
In Case 3b, \( \pm \bar{\sigma}_{\mu}' \) does not diminish as \( 1/\sqrt{N} \), and \( \pm \bar{s} \)
cannot be separated from \( \pm \bar{\sigma}_{\mu}' \).

3. RESULTS AND DISCUSSION
3.1. The average noise uncertainty estimate
It is now possible to evaluate the \( \pm 0.2 \) C uncertainty estimate of Folland, et al. [12],
who “estimated the two standard error (2\( \sigma \)) measurement error to be 0.4 \(^{\circ} \text{C} \) in any
single daily [land air-surface temperature] observation.” This estimate was not based
on a survey of sensors nor followed by a supporting citation. The temperature sensor
at each station will exhibit a unique and independent noise variance, and the context
in Ref. [12] provides that the \( \pm 0.2 \) C is from the estimated average variance of the
ensemble of variances of the individual surface station sensors entering measurements
into a global average surface air temperature. This estimated uncertainty thus falls
under Case 3, above.

Following this identification, the question next becomes whether the relevant
station sensor noise variances are stationary and of known magnitude. In general,
detailed examinations of errors in station histories have focused principally on
inhomogeneities due to instrumental changes and station moves [3, 9, 20-24], but have
not mentioned appraisals of station sensor variance. Reviews of time series quality control and homogeneity adjustments do not discuss sensor evaluation [7-10], and the methodological report of USHCN data quality [25] does not describe validation or sampling of noise stationarity in temperature sensors. The surface station sensor diagnostics, available in the online reports of the new USCRN National Climatic Data Center network, include standard deviations calculated from the twelve temperatures recorded hourly (http://www.ncdc.noaa.gov/crn/report; see the "Air Temperature Sensor Summary," under "Instruments"). But despite the set of ~8640 monthly standard deviations from individual CRN sensor data streams, which should give some measure of the magnitude and stationarity of variance, no extensive survey of station sensor variance is evident in published work.

The quality of individual surface stations is perhaps best surveyed in the US by way of the commendably excellent independent evaluations carried out by Anthony Watts and his corps of volunteers, publicly archived at http://www.surfacestations.org/ and approaching in extent the entire USHCN surface station network. As of this writing, 69% of the USHCN stations were reported to merit a site rating of poor, and a further 20% only fair [26]. These and more limited published surveys of station deficits [24, 27-30] have indicated far from ideal conditions governing surface station measurements in the US. In Europe, a recent wide-area analysis of station series quality under the European Climate Assessment [31], did not cite any survey of individual sensor variance stationarity, and observed that, "it cannot yet be guaranteed that every temperature and precipitation series in the December 2001 version will be sufficiently homogeneous in terms of daily mean and variance for every application."

Likewise, sensor variance was not mentioned in recent studies of data quality from surface stations in Canada [32, 33], where it was noted in 2002 that, "adjustments have only been carried out for identified step changes and the homogenized monthly temperatures have not been adjusted for artificial trends at this time." The authors stated further that, "The preferred methodology would be to develop procedures based on each cause of inhomogeneity. However, this would be a very site-specific task that would be nearly impossible to implement on a Canada-wide basis." Station evaluations for sensor variance are also not mentioned in the climate normals report of Environment Canada [34].

Thus, there apparently has never been a survey of temperature sensor noise variance or stationarity for the stations entering measurements into a global instrumental average, and stations that have been independently surveyed have exhibited predominantly poor site quality. Finally, Lin and Hubbard have shown [35] that variable field conditions impose non-linear systematic effects on the response of sensor electronics, suggestive of likely non-stationary noise variances within the temperature time series of individual surface stations (see Section 3.2.2).

These considerations indicate that an assumption of stationary noise variance in temperature time series cannot be presently justified, and that the assumption of random station errors is not empirically tenable. Therefore, the ±0.2°C estimate in Ref. [12] is the assessed ±σ°noise of Case 3b above, namely an adjudged assignment taken to represent the average uncertainty from an ensemble of surface station measurement
noise variances of unknown magnitude and stationarity. It does not represent the magnitude of random noise for any specific measurement, nor does it represent the noise variance of any specific sensor, nor is it an average of known stationary variances. Following Case 3b, the ±0.2 C estimate of Ref. [12] is ±σ'_{noise}, an adjudged constant average uncertainty that attaches to each surface temperature measurement and that must therefore be carried into a spatial average of N station measurements as ±σ'_{μ} = \sqrt{N \times σ^2_{noise}}/ (N − 1). This ±σ'_{μ} uncertainty does not decrement as 1/\sqrt{N} when calculating a mean, and will constitute a significant part of the total uncertainty in any global average surface air temperature index. Therefore the 1/\sqrt{N} noise reduction model in B06 is in error.

Clearly, a different statistical approach to uncertainty in air temperature averages is warranted; one that reflects the true empirical uncertainties. This approach is shown next.

3.2. An empirical approach to temperature uncertainty

T_{max} and T_{min} are typically measured many hours apart under physically opposing irradiance conditions. Although they may have an identical instrumental noise structure, they are experimentally independent measurements of physically different observables, each measured separately.

3.2.1. Uncertainty due to estimated noise

Following from Case 3b and the discussion in Section 3.1 above, the estimated per-measurement uncertainty ±σ'_{n} = ±0.2 C from Ref. [12] must enter as a constant applied to each temperature measurement. This value also represents the achievable uncertainty recommended by the World Meteorological Organization [36]. Using this value, and including normalization to a 30-year mean, the noise uncertainty in an annual anomaly is now stepwise calculated.

In order to maximally reduce the uncertainty, the annual temperature and the 30-year mean temperature were calculated directly, as though from individual measurements, as ̂T or ̂T = \frac{1}{N} \sum_{i=1}^{N} t_i, where ̂T is annual mean temperature, ̂T is a 30-year mean reference, t_i is an individual measurement, and N is the number of measurements (twice the number of days entering each mean). Applying the estimated average per-measurement uncertainty, ±σ'_{n} = ±0.2 C, the total noise uncertainty in any measurement average is

\[ ±σ'_{n} = \sqrt{\frac{N \times σ^2_{n}}{N − 1}} \]

(9)

where N is the number of measurements entering the mean. This calculation yields ±σ'_{n} = ±0.200 for both an annual mean temperature and a 30-year mean, and this uncertainty enters separately into each mean.

In calculating the uncertainty in an annual anomaly referenced against a multi-decadal mean, ΔT_{a} = ̂T − ̂T, the uncertainties in each mean value are combined in
The average noise uncertainty in the annual anomaly is then,
\[ \pm \sigma'_a = \sqrt{\left( \sigma' \right)^2 + \left( \sigma''_a \right)^2} = \sqrt{(0.2 C)^2 + (0.2 C)^2} = \pm 0.283 C, \]
where \( \pm \sigma' \) are the average noise uncertainties in the annual mean temperature, or the 30-year mean temperature, respectively. The \( \pm \sigma''_a \) represents the minimal noise-derived uncertainty in an annual temperature anomaly, referenced to a 30-year mean, for any given surface station when using the per-measurement estimated \( \sigma'_n = \pm 0.20 C \) of Folland, et al. [12].

It is also possible to obtain an annual anomaly by normalizing an annual temperature series to a fitted mean obtained by regression against a 30-year annual temperature time series. However, a regression mean introduces the uncertainty of the fit into the total uncertainty. This new uncertainty is the numerical estimated standard deviation (e.s.d.) of the fit, further scaled to reflect the reduced degrees of freedom induced by autocorrelation of the residuals [37]. The average uncertainty of each annual anomaly magnitude is then,

\[
\pm \hat{\sigma} = \sqrt{\left( \sigma_a' \right)^2 + \left( N \times (e.s.d.)^2 \right) / (N - \nu)},
\]

where \( (e.s.d.)_n \) is the numerical estimated standard deviation per point, \( N \) is the number of points, and \( \nu \) is the number of degrees of freedom lost through autocorrelation of the residuals.

Finally, in every case, a magnitude uncertainty, \( \pm s \), must also be included as part of the uncertainty in an annual anomaly. The magnitude uncertainty in an annual anomaly, \( \pm s_a \), can be estimated as \( \pm s_a = \pm s \times (\Delta T_a / \bar{T}) \), where \( \pm s \) is the magnitude uncertainty in a yearly average temperature, \( \Delta T_a \) is the average annual temperature anomaly, referenced to a 30-year mean, and \( \bar{T} \) is the average temperature for that same year. This uncertainty transmits the confidence that may be placed in an anomaly as representative of the state of the system.

3.2.2. Uncertainty due to systematic impacts on instrumental field resolution
The degraded instrumental resolution due to the systematic error from uncontrolled variables [38, 39] has apparently never found its way into any published assessment of the uncertainties in the global average surface air temperature index. The systematic measurement errors originating from the field exposure of the Min-Max Temperature System (MMTS), Automated Surface Observing System (ASOS), the Gill shield, and other commonly used electronic temperature sensors and shields have been investigated in excellent detail by Lin and Hubbard [35] and found to originate principally from solar radiation loading and wind speed effects. Other sources of error were enumerated as, "originating with the sensing element, analog signal conditioning, and data acquisition system [and] include the sensor interchangeability error, polynomial and linearization errors, self-heating error, voltage or current reference (excitation) error, total offset and drift in the amplifiers and ADC (associated
with stability), and lead wire error.” All these systematic errors, including the microclimatic effects, vary erratically in time and space [40-45], and can impose non-stationary and unpredictable biases and errors in sensor temperature measurements and data sets. These uncontrolled experimental variables degrade instrumental field resolution, and must be included in assessments of uncertainty in spatially and chronologically averaged temperatures.

Under ideal site conditions Hubbard and Lin recorded thousands of air temperatures using MMTS, ASOS, Gill, and other sensors and shields [13, 35, 42], and compared them to temperatures simultaneously measured using a calibrated high-resolution R. M. Young temperature probe with an aspirated shield. For each recorded temperature, the measurement rate was 6 min.\(^{-1}\) integrated across 5 min., reducing the random noise in each aggregated temperature measurement by \(1/\sqrt{30}\). The temperature data thus consisted primarily of a bias and a resolution width relative to the “true” temperature provided by the R. M. Young probe. Figure 1 shows the ideal day time field resolution envelopes of the MMTS, ASOS temperature sensors and Gill shield [13], fitted with the Gaussian

\[
a + b \times (1/\sqrt{2\pi}) \exp\left\{-[(1/2)(x - \mu/\sigma)^2]\right\},
\]

where \(a\) is a vertical offset, \(b\) is an intensity scaler, \(\sigma\) is the Gaussian (intensity)/\(e^{1/2}\) half-width (the standard deviation), and \(\mu\) is the Gaussian mean. The results of the fits are shown in Table 1.

Analogous fits to the 24-hour average data yielded, (sensor, \(\mu\) (C), \(1\sigma\) (±C), \(r^2\)): MMTS, (0.29, 0.25, 0.995); ASOS, (0.18, 0.14, 0.993), and; Gill, (0.21, 0.19, 0.979). These values are somewhat different from those originally reported in Ref. [13], which were not obtained from Gaussian fits.

The sensor evaluations were carried out under conditions of ideal siting and excellent maintenance [13]. Therefore, the field resolutions listed above approximate best achievable values and represent an attainable lower limit of instrumental resolution under field use conditions. Instrumental resolution by itself constitutes the minimum uncertainty in any temperature measurement, and the response of ideally sited and maintained sensors provides an empirical lower limit of field resolution.

<table>
<thead>
<tr>
<th>Table 1: Lower Limit Temperature Sensor Resolutions from Gaussian fits</th>
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<tbody>
<tr>
<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td>MMTS</td>
</tr>
<tr>
<td>ASOS</td>
</tr>
<tr>
<td>Gill</td>
</tr>
</tbody>
</table>
Figure 1. Daytime (a) and nighttime (b) resolution of: (o), MMTS; (□), ASOS, and; (Δ) Gill temperature sensors. The points were derived from Figure 2a,b of Ref. [13], using the program Digitizeit (www.digitizeit.de). The curves were normalized to unit area and each temperature bias relative to the R. M. Young probe was removed to yield a common mean of 0 C. The lines are Gaussian fits to the points. The ASOS and Gill data were vertically shifted 1 and 2 units, respectively, for clarity.

Temperature sensor resolution can, however, be significantly improved by application of a real-time empirical filtering algorithm to minimize the systematic error due to uncontrolled micro-climate variables [13, 46], as shown in Figure 2.
From the fits in Figure 2, the improved resolution of filtered MMTS data yields an uncertainty of ±0.093 C in daily mean temperature (cf. Figure 2, Legend). Using the equations gathered in Table 2, a minimum adjudged average ±0.1 C noise uncertainty and the ±0.093 C filtered resolution from a well-maintained MMTS sensor in an ideal site location, alone, yield a r.m.s. uncertainty in a per-station yearly temperature anomaly of \( \sqrt{(0.141)^2 + (0.132)^2} = ±0.193 \) C. However, the field resolution of surface station temperature sensors is not yet commonly improved using the Hubbard-Lin filter.

Figure 2. Gaussian fits to algorithmically filtered MMTS temperature resolution data, digitized as in Figure 4 and extracted from Ref. [46]. a. Daytime and b. nighttime, normalized to unit area and with the temperature bias again removed to produce a common mean of 0 C. The fit parameters are: a. \( \sigma = ±0.093±0.002 \) C, \( r^2 = 0.993 \), and; b. \( \sigma = ±0.029±0.001 \) C, \( r^2 = 0.987 \).
3.2.3. The lower limit uncertainty in an annual temperature anomaly

Appropriate statistics are now used to combine the average noise uncertainty of Section 3.2.1 and the ideal lower limit of systematic error from Section 3.2.2, into a composite lower limit of measurement uncertainty in surface station air temperature anomalies.

The equations used to propagate an appended per-measurement uncertainty into an annual anomaly, due to the entry of systematic error into field resolution, are analogous to those used to propagate the constant average noise uncertainty. One degree of freedom is lost in the statistical uncertainty mean because every determination of systematic error in a temperature data set is an average of the effects of uncontrolled variables. Each determination of field resolution uncertainty is also unique in terms of bias and width, because uncontrolled variables fluctuate in time and space. The idealized field resolutions for MMTS, ASOS sensors and the Gill shield referenced to a 30-year mean are shown in Table 2. For an MMTS sensor under ideal site conditions these equations yielded \( \pm \sigma_r = \pm 0.36 \, ^\circ\text{C} \), which represents a lower limit of resolution uncertainty from each such station entering into a global average anomaly. Table 2 also includes the analogous Case 3b average noise uncertainties from Section 3.2.1.

**Table 2: Uncertainty in an Annual Anomaly Due to Noise or Resolution\(^a\)**

<table>
<thead>
<tr>
<th>Uncertainty Eqn.(^b)</th>
<th>Noise ((\pm^\circ\text{C}))</th>
<th>Uncertainty Eqn.(^b)</th>
<th>Resolution ((\sigma_r, \pm^\circ\text{C}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_r = \sqrt{N \times \sigma_n^2 / N - 1} )</td>
<td>(0.100)</td>
<td>(\pm \sigma_r = \sqrt{N \times \sigma_n^2 / N - 1} )</td>
<td>MMTS: 0.254 | \text{ASOS: 0.137} | Gill: 0.189</td>
</tr>
<tr>
<td>(\sigma_n = \sqrt{(\sigma_n^2 + \sigma_r^2) / 2} )</td>
<td>(0.141)</td>
<td>(\pm \sigma_r = \sqrt{(\sigma_n^2 + \sigma_r^2) / 2} )</td>
<td>MMTS: 0.359 | \text{ASOS: 0.194} | Gill: 0.267</td>
</tr>
</tbody>
</table>

\(a\) Rows are, top: uncertainty in a yearly mean temperature; bottom: uncertainty in yearly anomaly referenced to a 30 year mean. \(b\) Uncertainty Equation. \(c\) 1\(\sigma\) (\(\pm^\circ\text{C}\); day, night): MMTS=(0.23, 0.17); ASOS=(0.16, 0.11), and Gill=(0.22, 0.12); see Table 1.

Figure 1 and Table 2 reflect average noise, and the resolution uncertainties currently expected from the ideal placement and maintenance of conventional surface station temperature sensors. For any one surface-station deploying a modern MMTS sensor, the minimal measurement uncertainty will be the average noise plus the ideal resolution uncertainties combined in quadrature [39, Section 5, 47]. From Table 2, for an MMTS sensor the total noise plus resolution lower-limit 1\(\sigma\) measurement uncertainty in an annual temperature anomaly referenced to a 30-year mean is

\[ \pm \sigma = \sqrt{(0.283)^2 + (0.359)^2} = \pm 0.46 \, ^\circ\text{C}. \]

The meaning of an ideal lower limit of measurement uncertainty provides that it is of lower magnitude than the uncertainty in each and all of the other homologous single-station measurements, worldwide. Thus, liquid-in-glass (LIG) thermometers in
Cotton Regional Shelters are reckoned to be of lower field resolution than the MMTS sensor [3, 41, 48]. Further, precision comparisons have shown that the systematic error introduced into surface station temperatures by the Cotton Regional Shelter is about twice that of the MMTS aspirated shield [42]. Thus, the ±0.46 C lower limit uncertainty of a modern MMTS sensor underestimates the uncertainty in the measurements from LIG thermometers in CRS shields that constitute the bulk of the 20th century global surface air temperature record.

The ±0.46 C lower limit of MMTS uncertainty is therefore applicable to every measurement in the global land surface record, because of the very high likelihood that it is of lower magnitude than the unknown uncertainties produced by surface station sensors that are generally more poorly maintained, more poorly sited, and less accurate than the reference sensors. The ideal resolutions of Figure 1 and Table 2 thus provide realistic lower-limits for the air temperature uncertainty in each annual anomaly of each of the surface climate stations used in a global air temperature average. This lower limit of measurement uncertainty for each surface station annual temperature anomaly is propagated into a global average as,

$$\pm \hat{\sigma}_{\text{total}} = \sqrt{\frac{\sum_{i=1}^{N} \hat{\sigma}_{i}^2}{N-1}},$$

(11)
to produce the total lower limit of uncertainty in a global temperature anomaly. Here $\hat{\sigma}_{i}^2$ is the lower limit mean noise plus resolution annual temperature uncertainty at the $i^{th}$ station, and N is the number of stations. For example, the lower limit of sensor uncertainty propagates into a global surface average air temperature anomaly as $\pm \sigma_{\text{global}} = \pm \sqrt{N \times (0.46 \, \text{C})^2/(N-1)} = \pm 0.46 \, \text{C}$, when, e.g., $N = 4349$ as in Ref. [11]. This uncertainty enters each anomaly in a global annual time series, and will be in addition to the commonly discussed uncertainties resulting from weather noise, step discontinuities, incomplete station coverage, land-use changes, siting artifacts [26], and albedo changes [49]. It seems likely that the new USCRN stations [50] will not significantly improve on the lower limit uncertainty any time soon [51].

### 3.2.4. The representative lower limit uncertainty in a global average air temperature anomaly time series

In independent calculations of global average surface air temperature anomalies, [52-54], the major source of uncertainty was assigned to incomplete station coverage, $2\sigma = \pm 0.07 \, \text{C}$ [55], with most of the remaining uncertainty assigned to the temporal inhomogeneity of temperature records [5]. These estimates of the global surface air temperature index did not include the instrumental uncertainties present in the surface station temperature measurements themselves, however. Therefore, the effect of the ideal lower limit uncertainty illustrated above on the reliability of the global average surface air temperature index is briefly considered below.
The uncertainties due to average noise and instrumental resolution in maritime temperature sensors remain to be evaluated and propagated into marine air temperature anomalies [11]. However, assessments of instrumental uncertainties in marine air and sea-surface temperatures have revealed evidence of significantly large systematic errors [56, 57], which both bias marine temperature measurements and imply an instrumental resolution degraded by uncontrolled environmental variables throughout the 20th century. Uncertainties in marine temperatures are thus not likely to be less than appraised here for land surface stations [4, 58, 59]. Therefore, the lower limit uncertainty in an MMTS land surface anomaly, $\sigma = \pm 0.46 \, ^\circ C$, can be credibly applied to the global land + ocean anomalies.

Figure 3 shows the global average surface air temperature anomaly index as compiled from surface and maritime meteorological stations and provided by the Goddard Institute for Space Studies, as updated on 18 February 2010. The lower limit $\pm 0.46 \, ^\circ C$ uncertainty in an annual surface anomaly is plotted on Figure 3 to illustrate a credible lower limit of uncertainty in the current surface air temperature anomaly series.

Figure 3. (•), the global surface air temperature anomaly series through 2009, as updated on 18 February 2010, (http://data.giss.nasa.gov/gistemp/graphs/). The grey error bars show the annual anomaly lower-limit uncertainty of $\pm 0.46 \, ^\circ C$. 
Figure 3 shows that the trend in averaged global surface air temperature from 1880 through 2000 is statistically indistinguishable from zero (0) Celsius at the 1σ level when this lower limit uncertainty is included, and likewise indistinguishable at the 2σ level through 2009. Thus, although Earth climate has unambiguously warmed during the 20th century, as evidenced by, e.g., the poleward migration of the northern tree line [60-62], the rate and magnitude of the average centennial warming are not knowable.

4. SUMMARY AND CONCLUSIONS

The assumption of global air temperature sensor noise stationarity is empirically untested and unverified. Estimated noise uncertainty propagates as $\pm \sqrt{N\sigma_n^2/(N-1)}$, rather than as $\pm \sigma_n/\sqrt{N}$. Future noise uncertainty in monthly means would greatly diminish if the siting of surface stations is improved and the sensor noise variances become known, monitored, and empirically verified as stationary.

The persistent uncertainty due to the effect of uncontrolled microclimatic variables on temperature sensor resolution has, until now, never been included in published assessments of global average surface air temperature. Average measurement noise and the lower limit of systematic sensor errors combined to yield a representative lower limit uncertainty of ±0.46 C in a 30-year mean annual temperature anomaly. In view of the problematic siting record of USHCN sensors, a globally complete assessment of current air temperature sensor field resolution seems likely to reveal a measurement uncertainty exceeding ±0.46 C by at least a factor of 2.

The ±0.46 C lower limit of uncertainty shows that between 1880 and 2000, the trend in averaged global surface air temperature anomalies is statistically indistinguishable from 0 C at the 1σ level. One cannot, therefore, avoid the conclusion that it is presently impossible to quantify the warming trend in global climate since 1880.

Finally, the relatively large uncertainty attending the global surface instrumental record means that the centennial temperature trend is not a precision target for validation tests of climate models. Likewise, the current surface instrumental record cannot credibly be used to train or renormalize any physically valid proxy reconstruction of paleo-temperature with sufficient precision to resolve any temperature difference less than at least 1 C, to 95% confidence. It is thus impossible to know whether the rate of warming during the 20th century was climatologically unprecedented, or to know the differential magnitude of any air temperature warmer or cooler than the present, within ±1 C, for any year prior to the satellite era. Therefore previous suggestions, that the rate or magnitude of present climate warming is recently or millennially unprecedented, must be vacated.

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