Calculating moist enthalpy from usual meteorological measurements.

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Abstract: This short article shows how to compute the moist enthalpy from usual meteorological measurements of dry temperature, air pressure and relative humidity. The result is used to add a plot of moist air enthalpy to the other near-live graphs shown by meteoLCD, the meteorological station of the LCD, Diekirch, Luxembourg

1. Sensible heat of dry air

The sensible heat of dry air is defined as H_a = Cp*T [ref. 3] with Cp usually taken as 1.005 when H_a is given in [kJ/kg] and temperature T in [°C].
Here we will use for Cp the following expression, valid for temperatures higher than 0 °C and lower than 60 °C, as given by PADFIELD [ref.2]

$H_a = 1.007*T - 0.026$ 0 °C < T < 60 °C [eq.1]

2. Heat content of water vapor at temperature T

The heat content of water vapor is the sum of the latent heat of vaporization and the sensible heat of water vapor:

$H_v = q*(L + 1.84*T)$ [ref. TET] [eq.2]

Where L = heat of vaporization = 2501 kJ/kg at 0°C
and 1.84*T = sensible heat of water vapor in kJ/kg

The sensible heat term of eq.3 (1.84*T) is very often considered negligible and omitted.

Note:

L is a function of temperature, becoming slightly smaller with increasing T; for values between 0°C and 50°C one can use the linear interpolation L(T) = 2502 – 2.378*T computed by the author from a table with enthalpy values given by YHCHEN [ref.4]:
The linear fit is excellent with R2 = 0.9998.
Combining eq.2 with L(T) gives:

H_v = q*(2502 – 0.538*T) with H_v in kJ/kg and T in °C [eq.3]

3. Total enthalpy of moist air

Total enthalpy of moist air is the sum of H_a and H_v:

H = H_a + H_v = (1.007*T -0.026) + q*(2502 – 0.538*T) [eq.4]

with H in kJ/kg, T in °C and specific humidity q in kg/kg

The problem with this formula is that the **specific** humidity q is usually not measured by a standard meteorological equipment which commonly measures **relative** humidity.

4. Finding q from measured dry bulb temperature, relative humidity and atmospheric pressure

PIELKE [ref.3] and the AOMIP website [ref.1] give the following formula for the specific humidity q:

$$q = \frac{0.622 * e_a}{p_a - 0.378 * e_a} \qquad \text{[eq.5]}$$

where e_a = vapor pressure in [Pa] and p_a = atmospheric pressure in [Pa].
Attention: p_a is the true air pressure, not the barometric pressure reduced to sea level!

Dividing numerator and denominator by e_a gives:

$$q = \frac{0.622}{\frac{p_a}{e_a} - 0.378} \qquad \text{[eq.6]}$$

Relative humidity is the fraction of water vapor pressure to saturated water vapor pressure, usually multiplied by 100 to give a percent value:

RH = 100* e_a/e_{sat} → e_a = RH/100*e_{sat}

There are many different formulas relating e_{sat} to temperature. We will use the expression given in AOMIP [ref.1] and valid up to 40°C:

$$e_{sat} = 10^{\left[\frac{0.7859+0.03477*T}{1+0.00412*T}+2\right]} \qquad \text{[eq.7]}$$

with saturated water vapor pressure e_{sat} in [Pa] and temperature T in °C.

Equations 4, 6 and 7 contain only T, RH and p_a, which are parameters measured by practically every standard weatherstation. Together they can be used to calculate the enthalpy of moist air by a single (albeit unwieldy) formula:

$$H = (1.007*T - 0.026) + (2502 - 0.538*T) * \frac{0.622}{\dfrac{p_a}{\dfrac{RH}{100} * 10^{\frac{0.7859+0.03477*T}{1+0.00412*T}+2}} - 0.378}$$

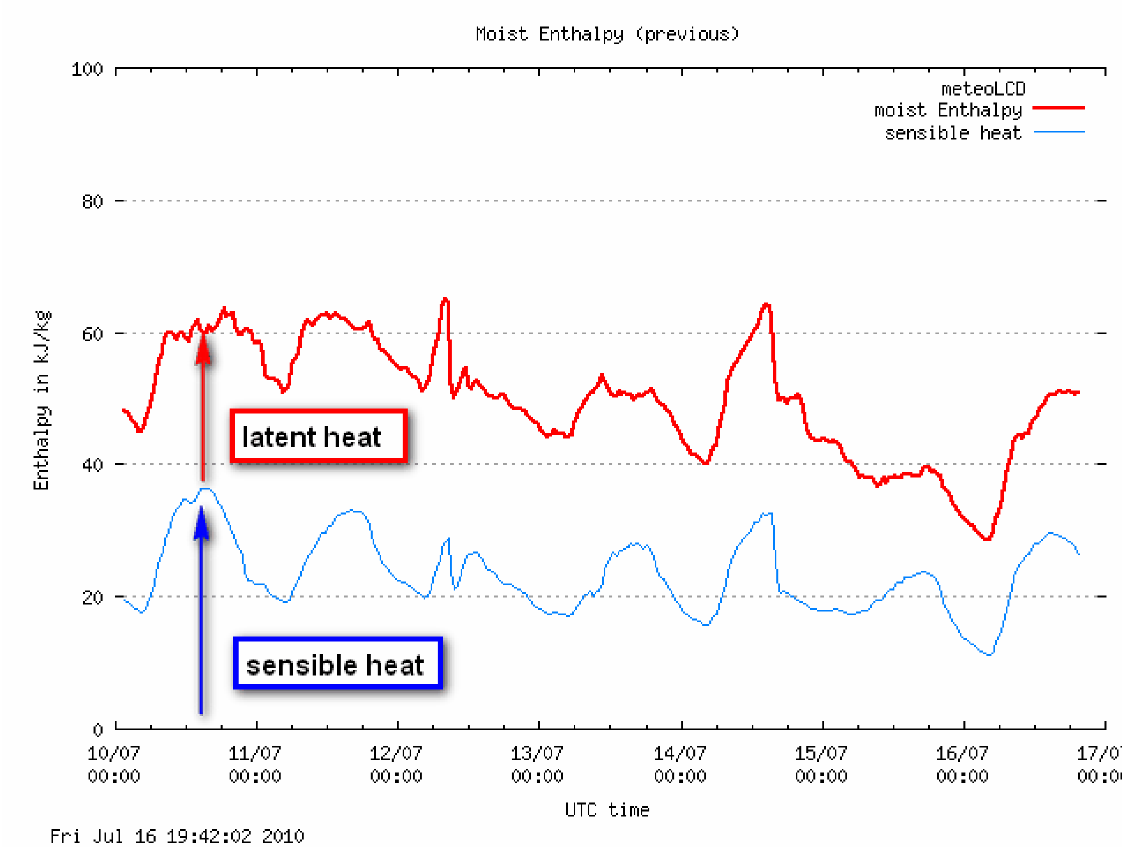
[eq.8]

This expression is valid for temperatures 0°C < T < 40°C. Units: H[kJ/kg], T[°C], p_a[Pa]

5. A practical example

The author has used eq.8 in GNUPLOT to display near-live plots of the moist enthalpy at meteoLCD, Diekirch, Luxembourg (see http://meteo.lcd.lu/today_01.html). The following figure shows the situation for the week from 10 to 16th July 2010. Sensible heat is shown by the blue bottom curve; the difference between the upper red curve (= moist enthalpy) and the blue curve corresponds to the latent heat.

Technisolve Software has a website with an online moist air calculator, which is very handy for a quick validation check of individual values:
http://www.coolit.co.za/airstate/airmoistobject.htm



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